A Local Force Model for Cardiac Dynamics Analysis Based on CT Volumetric Image Sequences*

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ABSTRACT

We present in this paper a local force model and its integration to a hierarchical analysis in the estimation of the left ventricle motion over a cardiac cycle. The local force model is derived from the dynamics of independent point masses driven by local constant forces over a short time period. A force field is assumed to remain constant over short periods of time driving independent point masses within a regional patch of the left ventricle surface from one time instant to another. The trajectory that minimizes the energy required to move the point mass from one surface to another is considered as the local displacement vector. This estimated trajectory takes into account surface constraints and previous estimations derived from the volumetric image sequences so that the point masses travel along smooth trajectories resembling the realistic left ventricle surface dynamics. This proposed model is able to recover the point correspondences of the nonrigid motions between consecutive frames when the surfaces and the initial conditions of left ventricle at consecutive time frames are given. The local force model is incorporated to a hierarchical analysis scheme[1] providing us a complete global and local dynamics of the left ventricle compared to only local kinematics analysis of previous approaches. Experimental results based on synthetic and real left ventricle CT volumetric images show that the proposed scheme is very promising for cardiac analysis.

1 INTRODUCTION

The techniques for acquiring images of beating heart have been advanced steadily for the last two decades.[2,3] As a result, left ventricle dynamics analysis based on various image sequences has received increasing attention for the last few years. In particular, the estimation of regional dynamics of the left ventricle surface can provide crucial information for heart decease diagnosis and monitoring. Although modern Cine CT scanners provide us with high resolution four dimensional data, the problem is very challenging because they are unable to offer regional features that can be tracked over the cardiac cycle. Furthermore, since the heart is a nonrigid object a complete description of its motion over a cardiac cycle requires the displacement information of each localized material element,[4] and the quantitative measurements of the local motion and deformation are often difficult due to the simultaneous presence of both global and local deformations for a beating heart. Such mixing of global and local motion and deformations have been observed by Potel and his colleagues.[5] Their observations suggested that, in addition to the widely believed regional motion and deformations, global movement of the left ventricle is also an integral part of the cardiac dynamics.

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There have been some success in applying image analysis-based approaches to the estimation of the left ventricle dynamics. The surface-based approaches[6–11] have attempted to characterize mainly the localized deformations of the left ventricle. Other nonrigid motion models potentially applicable to cardiac motion analysis include: finite element analysis[12–14] and deformable superquadrics.[15–17] However, most of these approaches based on nonrigid motion models have assumed that a set of corresponding landmarks are given or can be obtained. Some approaches overcome this problem by estimating the corresponding landmarks.[10,18,19,9] Duncan, et al.[18] proposes a bending energy model in the modelation of the left ventricle wall is a thin plate. The estimated plate bending energy is used to compute the best point correspondences between consecutive frames. One disadvantage of this approach is that the bending energy estimated via the surface curvature is sensitive to the parameterization of the surface, which change from frame to frame due noise and the nonrigid motion involved. Kambhamettu and Goldgof[19] used the Gaussian curvature as a surface descriptor in the recovery of the point correspondence under conformal nonrigid motion assumption. They choice of the Gaussian curvature as a surface descriptor, because it is invariant to the surface parameterization, allowing a more robust point estimation. However, the conformal motion assumption is a restrictive constraint and it is not appropriate for the analysis of non conformal motion like the one present in the left ventricle wall. Furthermore, both the bending energy and the Gaussian curvature assumptions are kinematics constraints limited to the estimation of very smooth motion vectors or small displacements of the left ventricle surface.

To overcome these problems, we propose in this paper a local force model. This model is a physics-based approach derived from the dynamics of independent point masses driven by local constant force over a short time period. A space variant force field is assumed to remain constant over short periods of time in a objet-centered coordinate system. The object-centered coordinate system allows the decomposition of the global and local motions, which make the local wall analysis simple and robust. This local force field drives the point masses within a regional patch of the left ventricle surface from one time instant to another. The trajectory that minimizes the kinetic energy required to move a point mass from one surface to another is considered as the local displacement vector. The estimated trajectory takes into account surface constraints and previous estimations derived from the volumetric image sequences so that the point masses travel along smooth trajectories resembling the realistic left ventricle surface dynamics. This proposed model is able to recover the point correspondences of the nonrigid motions between consecutive frames when the surfaces and the initial conditions of left ventricle at consecutive time frames are given.

In this research the hierarchical analysis approach, described at Tamez-Pena and Chen,[20] is used to obtain a coarse estimation of the left ventricle surface and to estimate the global motion. The parameterization of the estimated surface is used to guide the point correspondence recovery. Furthermore, the decomposition of the motion in global and local motions makes the approach more robust to local surface variations. Thus, the force model incorporated to the hierarchical analysis provides us with a complete global and local dynamics of the left ventricle compared to only local kinematics analysis of previous approaches. Furthermore, our approach is independent of the surface parameterization and curvature making it is more robust and simple to compute.

The paper organization is as follows. First, we are going to present the local force model and its application to the point correspondence recovery, based in the minimization of the kinetic energy. Then, our implementation of the hierarchical analysis, and the incorporation of the local force model to it, related to the left ventricular analysis, will be presented. Finally, initial experimental results based on synthetic and real left ventricle CT volumetric images will be presented to show the advantages of the local constant force model over existing approaches.

2 LOCAL FORCE MODEL

The point correspondence recovery developed in this paper is based in the dynamics of a point mass driven by a constant force over short periods of time. This section describes and justifies the local force model for any surface patch subject to external and internal forces and its application to the tracking of non-rigid motion, based on its kinetic energy.

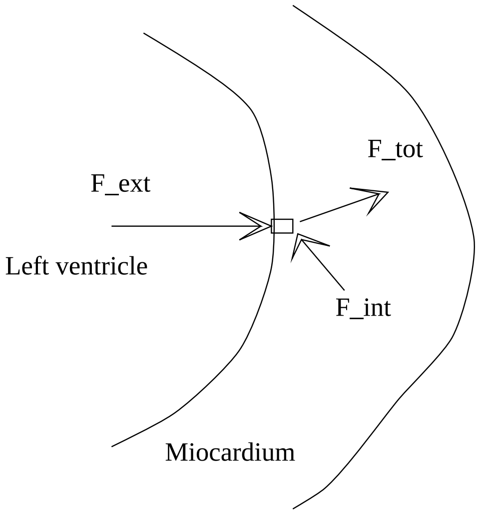

Figure 1: Local Force Model

First consider a time varying deformable object boundary $S(t)$, and an infinitesimal surface patch at $S(t)$, as illustrated in Fig. 1. This patch is subject to local time-varying internal ($F_{int}(t)$) and external forces ($F_{ext}(t)$). The sum of the internal an external forces along the patch mass (m) define its trajectory ($R(t)$). The dynamic equation that describe the infinitesimal patch behavior is

$$\sum F_{int}(t) + \sum F_{ext}(t) = m\frac{d^2R(t)}{dt^2}. \tag{1}$$

Now, assume that the total force, given by sum of the internal and external forces, varies slowly over time. Then, a good approximation for the total force can be given by piecewise sum of constant forces. That is, the total force is assumed constant over a short period of time T. Then equation (1) is transformed to a piecewise approximation of the mass point dynamics. Thus, for any time interval the particle dynamics is,

$$K(n) = m\frac{d^2R(t)}{dt^2}, \qquad nT < t < (n+1)T, \tag{2}$$

where n is an integer. Finally, the solution to this equation has a quadratic form between the $nT < t < (n+1)T$ interval, and it is given by

$$R(t) = A(n) + B(n)t + C(n)t^2, \qquad nT < t < (n+1)T. \tag{3}$$

The above development represents a quadratic spline approximation of the real trajectory of a material patch R in a non-rigid object boundary. Although the constant vectors $A(n), B(n) and C(n)$ can be determined from the boundary conditions its interpretation is not straight forward. However, rewriting equation (3) as:

$$R(t) = A'(n) + B'(n)\Delta t + C'(n)\Delta t^2, \qquad nT < t < (n+1)T, \tag{4}$$

where $\Delta t = t - nT$. The new parameters A', B' and C' have a clear meaning. At sample points $t = nT$, $\delta t = 0$ therefore $A'(n) = R(nT)$, and $B'(n) = \frac{dR(nT)}{dt}$, the sampled position and the estimated velocity respectively. While, at $t = (n+1)T$, $\Delta t = T$ thus

$$C'(n) = \frac{R_i((n+1)T) - A'_i(n) - B'_i(n)T}{T}$$

, so $C'(n)$ is function of the next sample position, the actual sample position and the estimated velocity.

2.1 Point Correspondence Recovery

The local force model can be used to justify a spline trajectory of any known point between the sampled surfaces $S(nT)$, but usually the point correspondence between sampled surfaces is unknown. That is, we don't know the next sample position $R((n+1)T)$ at $S((n+1)T)$. Thus, the real problem is to find the best point in the next surface $S((n+1)T)$ that meets some criterion. Duncan[21,22] and Goldgof[19] have proposed some shape based approach to find the best point correspondence between surfaces, but this require the computation of some local surface properties, which are prone to noise making them not very realiable. Here we propose to use the local force model to estimate some trajectories and select the one which minimize the energy required to move the point from one surface to the other. This new approach does not require the estimation of some local surface properties, making it less sensitive to computational errors.

2.1.1 Kinetic Energy

The kinetic energy change used to move one point mass m from point r_1 to point r_2 is given by

$$U = \int_{r_1}^{r_2} F \cdot dr = \frac{1}{2} m (v_2^2 - v_1^2), \tag{5}$$

where v_1 and v_2 are the initial and final velocities of the mass m, and F is the total force. Thus, the point correspondence recovery is given by selecting the trajectory which requires a minimum change in the particle kinetic energy, which is equivalent to the minimization the absolute difference on the square velocities. For the local force model, i.e.

$$\min |V((n+1)T)^2 - V(nT)^2|, \tag{6}$$

where $V(nT) = B'(n)$ and $V((n+1)T) = B'(n) + C'(n)T$. Thus, the search of the point with minimum energy involves the evaluation of a very simple equation, where $C'(n)$ is the only parameter who is function of the point in the next surface. Therefore the energy minimization is function o the initial velocity and the final posiiton, which simplifies the search of the point with minimum energy.

3 Left Ventricular Analysis

The application of the local force model to the analysis of the left ventricular motion, requires an accurate description of the left ventricle surface and overall an appropriate reference frame. A correct selection of the reference frame is very important for the proposed point tracking algorithm because it is not a shape matching scheme, thus all global motion should be estimated and removed prior any attempt to recover the point correspondence between frames. The shape description and and the selection of the reference frame are accomplished by a hierarchical analysis[23,20]; but first, the left ventricle must be extracted from 3D image sequences. This section describes the left ventricle extraction, the application of the hierarchical analysis and the incorporation of the local force model to the functional description of the left ventricle.

3.1 CT images and the left ventricle wall segmentation

Our left ventricular motion analysis is based on the tracking of the left ventricle surface. Furthermore, our local force model requires, that the left ventricle surface, be sampled in a balanced spatial and temporal resolutions. Among all imaging modalities of the heart, CT volumetric image sequences of heart have been considered able to meet these standards. The volumetric image sequence used in this research was obtained from the unique Dynamic Spatial Reconstructor (DSR) operated by Mayo Clinics.[2] Compared with the commercially available

Picker Fastrac, or Imatron scanners, the DSR scanner has functional flexibility in that spatial, temporal, and contrast resolution can be adjusted to favor one aspect of resolution over the other. This flexibility facilitates basic research applications.

In a typical volume of the DSR, the left ventricle is included in a high intensity region which would also includes the left atrial chamber and aorta. Although there are valves separating the left ventricle chamber from the left atrial chamber and aorta, the valves of canine heart, which has been used in this basic research rather that the human heart, are only of the order of 1 mm thick and their visibility in the volume is diminished by the partial volume effect and resolution limitation of DSR scanner.[24] Furthermore, the valves open and close alternatively during a cardiac so that whether or not they appear in an acquired image would also depend on the timing of image acquisition. Therefore the left ventricle often appears connected with the left atrial chamber and aorta in the acquired volumetric images.

Overall, the intensity of the left ventricle is much brighter than the myocardium. However, the intensity distribution of the left ventricle chamber is not uniform due to the uneven distribution of the contrast agent. The nonuniform distribution of the contrast agent along with the problem of the atrial chamber and aorta makes the extraction of the left ventricle from DSR data a very difficult problem. Although, there have been some attempts[25,26] to automatically or semi automatic extract the left ventricle form the volumetric data they are not very reliable. Therefore in this study, we used the 3D volume data obtained via a manual extraction of the left ventricle from the DSR original data because it is reliable and it allows us to concentrate on the motion analysis of the left ventricle.

3.2 Hierarchical Analysis

The left ventricle motion estimation can be simplified by following the hierarchical analysis proposed by Chen.[23] This hierarchical analysis is an efficient global to local scheme, which helps to simplify the study of complex objects such as the left ventricle. The hierarchical analysis starts with the selection of an appropriate coordinate system and an analysis of the rigid global properties of the left ventricle, follow by a coarse shape description. Finally a local shape and motion analysis is performed.

3.2.1 Global transformation

The left ventricle shape is not symmetric but can be coarsely modeled by a simple bended tapered ellipsoid. The origin of the tapered geometric objects differs from its mass centroid. Then, the estimation of the object-centered coordinate system must be independent of tapering deformations. In the left ventricle case, we observe that the long axis, which is a curve in the 3D space that lies approximately in a plane that joins the base with the apex, could be used to estimate this centroid because it is free of tapering deformations. Another advantage of using the long axis is that its bending plane would give us information of the left ventricle global rotation. Furthermore, the long axis can be fitted to a bending function which is used to unbend the left ventricle surface points. The estimation of the bending plane, and the bending function, requires the estimation of the long axis. In our case of left ventricle chamber, represented by the digital surface obtained from manual segmentation, this axis is estimated by using the slice centroids

$$\begin{bmatrix} \overline{x}_i \\ \overline{y}_i \\ \overline{z}_i \end{bmatrix} = \begin{bmatrix} 1/N_i \sum_{j=1}^{N_i} x_{i,j} \\ 1/N_i \sum_{j=1}^{N_i} y_{i,j} \\ z_i \end{bmatrix}, \tag{7}$$

where N_i are the number of boundary points on slice i.

The ellipsoid parameters and the rest of the global transformations can be easily estimated if we use the long axis centroid,. The main axis centroid do not carry information over the shape volume so it is a better estimator of the model origin. Thus, the origin of the new reference system is given by:

$$\begin{bmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{bmatrix} = \begin{bmatrix} 1/M \sum_{i=1}^{M} \overline{x}_i \\ 1/M \sum_{i=1}^{M} \overline{y}_i \\ 1/M \sum_{i=1}^{M} \overline{z}_i \end{bmatrix}, \tag{8}$$

where M is the number of slice centroids in the long axis.

The long axis is bended in a plane. This plane can be estimated from the the slice centroids via the computation of the scattering matrix.[23] The scatter matrix is computed using the distance of the long axis points to the left ventricle centroid:

$$P_i = \begin{bmatrix} \overline{x}_i - \overline{x} \\ \overline{y}_i - \overline{y} \\ \overline{z}_i - \overline{z} \end{bmatrix}, \tag{9}$$

and the scattering matrix is defined as:

$$[S] = \sum_{i=1}^{M} P_i P_i^T. \tag{10}$$

Now the orientation of the normal of the best fitting plane is given by the eigenvector associated with the smallest eigenvalue of the scatter matrix. At this point we have all the information to transform the surface points from the world coordinate system to the object-centered coordinate system. The new origin is given by the long axis centroid and the orientation is computed using the eigenvectors of the scatter matrix. Where the x-axis is defined as the normal to the best fitting plane and the y and z axis are estimated by the other two eigenvectors of the scatter matrix which are orthonormal to each other.

The centroid and the scattering matrix can be used to compute the global motion parameters. The centroid of each slice is used to compute the translation vector:

$$T = \begin{bmatrix} \overline{x}_m \\ \overline{y}_m \\ \overline{z}_m \end{bmatrix} - \begin{bmatrix} \overline{x}_{m+1} \\ \overline{y}_{m+1} \\ \overline{z}_{m+1} \end{bmatrix}.$$

where $(\overline{x}_m, \overline{y}_m, \overline{z}_m)$ are the centroids at frame m. The eigenvectors of the scattering matrix (10) are used to estimate the rotation matrix between frames m and $m+1$ via:

$$R = [\, e_{x,m} \quad e_{y,m} \quad e_{z,m} \,][\, e_{x,m+1} \quad e_{y,m+1} \quad e_{z,m+1} \,] \tag{11}$$

where $(e_{x,m}, e_{y,m}, e_{z,m})$ are the eigenvectors of the scattering matrix at m.

3.2.2 Unbending transformation

The left ventricle complex bended shape will be modeled by a bended tapered ellipsoid. Therefore, the recovery of the tapered ellipsoid parameters can done by finding a transformation that unbends the left ventricle shape. We follow an extension of a simple 2D conformal bending deformation as shown in figure 2, where only one line on the 2D space suffers an isometric deformation, i.e. it preserves it length, while the other points follows a conformal mapping according the a deformation function. This conformal bending can be extended to the 3D world, where now a single a plane is isometrically bended to a bending function, and the inner and outer points follow the conformal mapping.

If the long axis is modeled by a function $f_u(z)$ that lies in the y, z plane, then the conformal transformation of each point (x, y, z) in the object centered left ventricle surface to a point (x', y', z') in the unbended space can be

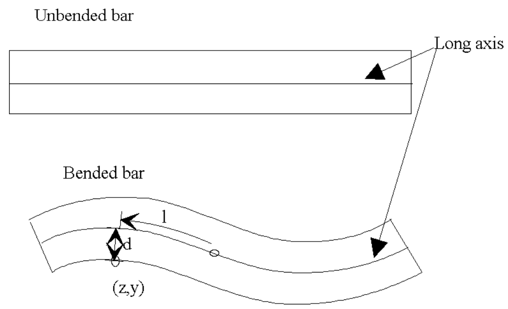

Figure 2: Bending model

written as:

$$z'(z_i) = \int_0^{z_i} \sqrt{1 + \left(\frac{df_u(z)}{dz}\right)^2} dz \tag{12}$$

and

$$y' = \sqrt{(y - f_u(z_i))^2 + (z - z_i)^2}, \tag{13}$$

$$x' = x \tag{14}$$

where $(f_u(z_i), z_i)$ is the intersection point of the shortest line connecting a surface point to the long axis plane $f_u(z)$, and it is the root of the following equation:

$$h(z_i) = -y_s - \left[\frac{df_u(z)}{dz}\right]^{-1}_{z=z_i} (z_s - z_i) + f_u(z_i) = 0 \tag{15}$$

This transformation maps each surface point (x, y, z) from the isometric coordinate system to (x', y', z') in the unbended coordinate system, and the x coordinate is unaffected by such transformation. Such transformation converts the original left ventricle surface data into a new reference system in which the modeling primitives can be directly fitted to the left ventricle surface data. Without such transformation, the original surface data derived from the left ventricle chamber may not fit well a tapered ellipsoid, since the modeling primitives are a family of symmetric surfaces while the real left ventricle surfaces exhibit some bended characteristics. In our study we assume a simple polynomial, i.e.

$$f_u(z) = a_o + a_1 x + a_2 x^2 + a_3 x^3 + \ \ldots \quad .$$

for the bending function $f_u(z)$. Figure 3 shows the left ventricle surface before and after the unbending transformation.

3.2.3 Tapered ellipsoid space

Once we have transformed each surface point to the unbended surface points we can proceed to fit these points to a tapered ellipsoid. An ellipsoid can be defined in vector form as follows:

$$S(\theta, \phi) = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_x \cos(\theta)\cos(\phi) \\ a_y \cos(\theta)\sin(\phi) \\ a_z sin(\theta) \end{bmatrix}, \tag{16}$$

where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $-\pi \le \phi \le \pi$. Parameters θ and ϕ correspond to the latitude and longitude angles, respectively, expressed in the unbended object-centered spherical coordinate system. Angle ϕ lies in the $x - y$

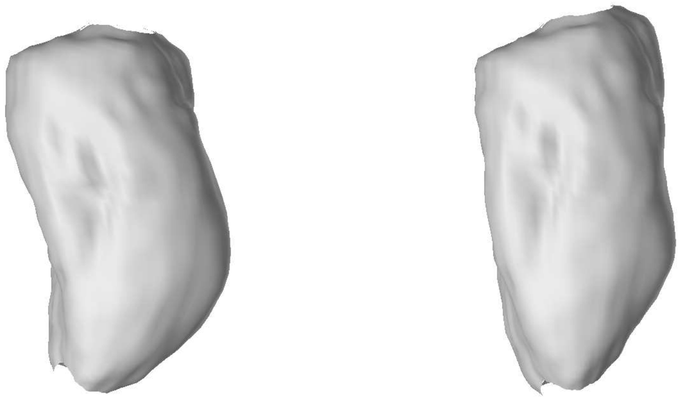

Figure 3: Left, original left ventricle surface for frame 1 of the DSR data set. Right, left ventricle surface after the unbending transformation

plane, while θ is the angle between the vector $S(\theta,\phi)$ and the $x-y$ plane. Scale parameters a_x, a_y, a_z define the size of the ellipsoid in the x, y and z directions, respectively. We can extend the ellipsoid abilities to model the left ventricle shape by introducing a tapering deformation. The tapering deformation represent a scale change in one axis. For a z-axis tapering deformation:

$$X = f_x(z')x', \quad Y = f_y(z')y', \quad Z = z', \tag{17}$$

where $f_x(z')$ and $f_y(z')$, the tapering functions, are usually piecewise linear functions of z' according to*a priori* knowledge of the left ventricle. In general, the tapering deformation for left ventricle modeling can be a simple linear function of z', i.e.

$$f_x(z') = k_x z' + 1, \quad f_y(z') = k_y z' + 1,$$

where k_x and k_y are the tapering constants along the x' and y' axes, respectively.

With the addition of the tapering deformation to the ellipsoid model, our coarse shape model is now complete which is now defined as:.

$$s'(\theta,\phi) = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} f_x(z)a_x\cos(\theta)\cos(\phi) \\ f_y(z)a_y\cos(\theta)\sin(\phi) \\ a_z sin(\theta) \end{bmatrix}. \tag{18}$$

The tapered ellipsoid fitting can be stated as a non-linear optimization problem, using the inside-outside function, derived from the ellipsoid shape primitives, and can be written as:

$$f(x,y,z) = \left(\left(\frac{x}{f_x(z)a_x} \right)^2 + \left(\frac{y}{f_y(z)a_y} \right)^2 \right) + \left(\frac{z}{a_z} \right)^2, \tag{19}$$

where if $f(x_o, y_o, z_o) = 1$, then (x_o, y_o, z_o) is on the surface; if $f(x_o, y_o, z_o) < 1$, the point is inside the surface, and if $f(x_o, y_o, z_o) > 1$, the point lies outside the superquadric surface. The objective function for the optimization problem can be defined as:

$$minimize: \sum_{i=1}^{n} |f(x,y,z) - 1|^2, \tag{20}$$

where the summation is over all the surface points.

Once we have fitted the left ventricle to a tapered ellipsoid, we are able to transform every surface point on the unbended space to a ellipsoidal coordinate system. The coordinate transformation is given by

$$\begin{bmatrix} r \\ \phi \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{x'^2 + y'^2 + z'^2} \\ \tan^{-1}(\frac{y'/(f_y(z')a_y)}{x'/(f_x(z')a_x))} \\ \tan^{-1}(\frac{z'/a_z}{\sqrt{(y'/(f_y(z')a_y))^2 + (x'/(f_x(z')a_x))^2}}) \end{bmatrix}. \tag{21}$$

This transformation allows us to represent the left ventricle surface as parameters of θ, ϕ. That is, the left ventricle surface points define a function $S(\theta, \phi)$ in the ellipsoidal space. The parametric surface representation is very helpful in the tracking of the left ventricle wall.

3.3 Left ventricle local motion tracking

The hierarchical transformations maps the surface points from the world image space into a bended tapered ellipsoidal space. In the case of the local motion analysis, the more important transformation is the first one, which is used to get a reference frame free of global motion. Therefore we can concentrate on the analysis of the local motions and deformations. The unbending transformation and the ellipsoidal mapping are used to transform the 3D search space into a 2D parametric search space, which it is very simple to search.

The proposed point correspondence recovery approach described in section 2 based on the local force model and the kinetic energy constrain is the basic tool for the local motion analysis of the left ventricle. For each point in the first surface $S(n)$ at (θ_i, ϕ_i) a 2D small search window in the neighbor of (θ_i, ϕ_i) is set at the next surface $S(n+1)$. At each point, on the next surface, the kinetic energy is computed based on the local force model and the point with the minimum kinetic energy is selected. Some times there are more than one point with minimum energy. In that case we decided to select all of them and choose the average position as the corresponding next point in the surface. A better approach is to add more constrains in the selection of the point correspondence, like the internal energy; but at this time we have not included any internal energy considerations.

The computed kinetic energy is function of the initial velocity vector at (θ_o, ϕ_o), this velocity is function of the previous estimated velocities, but for the first frame we have to relay on the our *a priory* knowledge of the left ventricle to set the initial velocity. Our approach was to start the analysis of the left ventricle when it is in its end-systole state. This allow us to assume that the initial velocity of any point in the surface is zero. This assumption, although is not true for every point in the left ventricle wall, it is close to reality.

3.3.1 Velocity field smoothing

The constant local force assumption will produce some errors in the point correspondence and the final velocity. These errors will propagate to further point estimations, because the point estimation and the final estimated velocity, $R((n+1)T)$ and $V((n+1)T)$ respectively, are functions of to the initial estimated velocity $V(nT)$. The errors in the estimated velocity will cause inconsistent motion vectors. Therefore, to make the estimations more consistent we decided to smooth each estimated velocity vector field, before the estimation of the next point correspondence. Each velocity vector field is smoothed in the parametric space (r, θ, ϕ) following a simple average spatial filter:

$$V(nT, \theta, \phi) = \int\int V(nT, \theta - u, \phi - v) h(u, v) du dv, \tag{22}$$

where $h(u, v)$ is the average filter, and $V(nT, \theta, \phi)$ is the estimated velocity filed. The smoothing of the vector field will help in the generation of more consistent motion vectors.

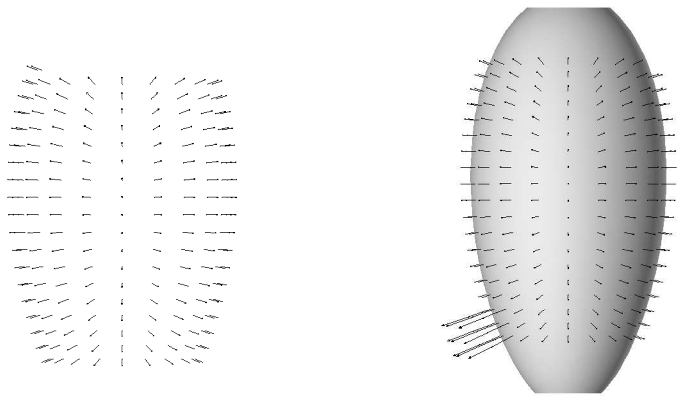

Figure 4: Left, point trajectory from frame 1 to 2 in ellipsoid expansion. Right, point trajectories from frame 1 to 4.

4 Experimental Results

Using the methodology described above, we performed two initial set of experiments. The first experiment consisted in the point correspondence recovery in a simulated ellipsoid. The second experiment consisted in the motion analysis of real set of data from the DSR.

4.1 Synthetic Data

The first experiment was performed over synthetic data and was designed to test the ability of the local force model to recover the point correspondence. An ellipsoid was deformed unevenly and a set of four consecutive images was generated. Figure 4 shows the evolution of the surface at the four samples. The arrow flows show the estimated points motions over time, where each error representing the one point movement between a pair of frames at n and $n+1$. Note the smooth flow of the point through the evolution of the deformation, which it is desired.

4.2 Real Data

The real experiment was performed over a set of 4D data from the DSR. The data set consisted on 16 manually segmented frames over the cardiac cycle. Figure 5 shows the estimated point displacement at different time instants, where the arrows show the local displacement of one point from frame n to $n+1$. The real experiments showed the limitations of the local force model at points where the surface undergoes large deformations, nevertheless the estimated displacement vectors at other points are consistent.

5 CONCLUSIONS

In this paper, we have presented a physical based approach for the analysis of local deformations in nor-rigid bodies. This model was based on the dynamics of point masses subject to constant forces over short periods of time and it is was applied to the point correspondence recovery. The point correspondence was set for the

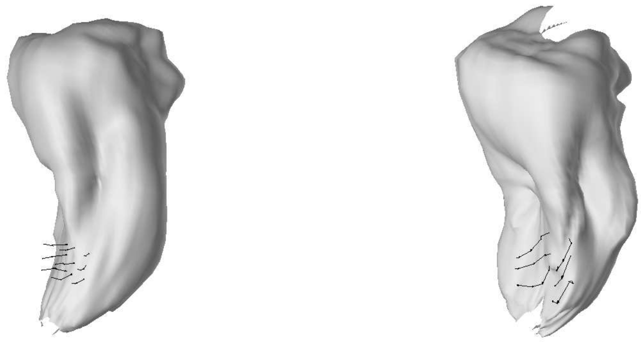

Figure 5: Left, some left ventricle point trajectories from frame 1 to 6. Right, point trajectories from frame 9 to 14

trajectory with minimum kinetic energy. Our new approach incorporated to the hierarchical analysis proposed at[23] was successfully applied to the study of the local dynamical function of the left ventricle of the heart from 4D data. Furthermore a complete functional analysis can be done under the hierarchical analysis scheme, where global motions and bending properties are extracted, being an important part of the heart dynamics. The point correspondence search can be improved with the incorporation of some internal energy constrains. Thus the future work includes the incorporation of the internal energy as integral part of the local force model.

6 Acknowledgment

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