Cardiac Dynamic Analysis Using Hierarchical Shape Models and Gaussian Curvature Recovery: An Integrated Approach

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ABSTRACT

We present in this paper a scheme to analyze the left ventricle motion over a cardiac cycle through the integration of the hierarchical surface fitting and the point correspondence estimation. The hierarchical surface fitting is a coarse-to-fine analysis scheme and has been successfully applied to cine-angiographic cardiac images.[1] In this study, the hierarchical surface fitting and motion analysis is applied to a set of CT images with real volumetric nature. We also incorporate an additional global deformation, long axis bending, into the shape model to reflect the curved nature of the left ventricle long axis. With the dense volumetric data, we are able to implement higher order spherical harmonics in the analysis of the local deformations. The fitted surface allows us a complete recovery of the Gaussian curvature of the shape. The estimation of the point correspondence is accomplished through the analysis of the first fundamental form and the Gaussian curvature computed from the fitted shape assuming conformal motion. The overall coarse-to-fine hierarchical analysis and the parametric nature of the fitted surface enable us to compute of the Gaussian curvature analyticaly and gain a clear and complete description of the left ventricle dynamics based on the shape evolution over the cardiac cycle. Results based on a set of CT data of 16 volumes show that this hierarchical surface fitting and motion analysis scheme is promising for cardiac analysis.

Keywords: cardiac motion, surface fitting, motion analysis, point correspondece recovery, deformable models, hierarchical models, conformal motion, Gaussian curvature.

1 INTRODUCTION: CARDIAC DYNAMICS ANALYSIS

The techniques for acquiring images of beating heart have been advanced steadily for the last two decades.[2,3] As a result, left ventricle dynamics analysis based on various image sequences has received increasing attention for the last few years. There have been some success in applying image analysis-based approaches to the estimation of the left ventricle dynamics. The surface-based approaches[4,7] have attempted to characterize mainly the localized deformations of the left ventricle. Other nonrigid motion models potentially applicable to cardiac motion analysis include the modal finite analysis,[8] deformable superquadrics,[9] and the combination of the both.[10] However, all approaches based on nonrigid motion models have assumed that a set of corresponding landmarks can be obtained.

Modern Cine CT scanners provide us with high resolution four dimensional data which contain tremendous amount of information about the cardiac dynamics. However, quantitative regional features cannot be easily tracked within the cardiac cycle using such data.[4] There have been some attempts to solve this problem with initial success.[7] Conformal nonrigid motion has been assumed to model the local left ventricle wall as a thin plate in which the bending energy is computed to match the best point correspondence between consecutive frames.[7] This approach has been applied to CT volumetric image data to obtain reasonable estimation of local correspondences. One disadvantage of this approach is that the bending energy obtained is very sensitive to the parametrization of the surface, since such parametrization changes from frame to frame due to noise and nonrigid motion involved. A potential solution to the problem of thin plate bending model is to use Gaussian curvature as a surface descriptor, because Gaussian curvature is invariant to the surface parametrization under the assumption that the surface undergoes conformal nonrigid motion.[5] The invariant property of the Gaussian curvature can be used to recover the point correspondence of the left ventricle dynamic surface if the surface can be successfully reconstructed from the CT volumetric data for consecutive frames. The existing approach of surface reconstruction within the framework of Gaussian curvature used a quadratic surface reconstruction based on local data points. Such localized surface reconstruction lacks the ability to characterize the overall dynamics, both global motion and local deformation, of the left ventricle.

This paper attempts to develop an integrated approach for a complete cardiac dynamic shape analysis based on hierarchical surface fitting and the Gaussian curvature estimation. It is true that the ischemic heart diseases are often regional, and the measure of local motion and deformation of the left ventricle over a cardiac cycle provide important information in cardiac functioning. However, the quantitative measurements of the local motion and deformation are often difficult due to the simultaneous presence of both global and local deformations for a beating heart. Such mixing of global and local motion and deformations have been observed by Potel and his colleagues.[11] Their observations suggested that, in addition to the widely believed regional motion and deformations, global movement of the left ventricle is also an integral part of the cardiac dynamics. Therefore, in order to more accurately measure the local motion and deformation, the estimation and compensation of the global motion and deformation is necessary. We expect that a complete cardiac dynamic analysis, both global and local, will provide a more accurate estimation of the cardiac motion and deformation.

2 CT VOLUMETRIC DATA AND 3D ANALYSIS

Among all types of images acquired for the analysis of heart motion and deformations, CT volumetric image sequences of heart have been considered able to provide balanced spatial and temporal resolutions. The volumetric image sequence used in this research is obtained from the unique Dynamic Spatial Reconstructor (DSR) operated by Mayo Clinics.[2] Compare with the commercially available Picker Fastrac, or Imatron scanners, the DSR scanner has functional flexibility in that spatial, temporal, and contrast resolution can be adjusted to favor one aspect of resolution over the other. This flexibility facilitates basic research applications.

Until recently, such volumetric image sequences have been used mainly for the extraction of motion-related quantities to indirectly infer the dynamic information of the heart.[2,12] Even though the heart undergoes nonrigid motion and deformation in 3D space, many existing methods of quantitative cardiac motion analysis are actually based on 2D cardiac image sequences, i.e., the projections of beating heart onto fixed 2D planes, or 2D sections.[13] These 2D methods are therefore not directly applicable to the volumetric data generated by DSR. To take full advantage of the high resolutions in both spatial and temporal domains of these volumetric CT cardiac image sequences, researchers have started to develop 3D motion and deformation estimation algorithms for cardiac dynamic analysis.[14–16] These 3D methods are potentially able to overcome many problems associated with the 2D approaches, such as the simplified assumption of radial thickening.[17] In particular, the errors in 2D thickness estimation may become crucial when the spatial orientation of the cross sections of the heart deviate significantly from the short axis position. The 3D data used in this research is the 3D surface points extracted from the original data by manual segmentation of the 3D volumetric data generated by the DSR.

3 HIERARCHICAL MODELING

The left ventricle chamber can be considered as a deformable object whose shape is undergoing nonrigid motion and deformations. For a general nonrigid motion, each element within an object often requires individual characterization. However, in the case of the beating heart, the shape of the left ventricle chamber changes regularly and periodically over each cardiac cycle. Such shape changes are closely related to, if not completely determined by, the dynamics of the heart. Naturally, dynamics analysis of the left ventricle can be accomplished through surface modeling with appropriate parameterization to avoid the difficult task of characterizing the dynamics of each local element. In fact, many cardiac researchers have implemented various shape modeling algorithms for left ventricle dynamics analysis, ranging from the simple cylinder segment models[18] to the more complicated ellipsoid surface models.[19] The success of existing surface models has been limited due to the oversimplified geometry, which is in sharp contrast to the complex nature of the left ventricle shape and dynamics. A realistic surface modeling, which includes both global and local deformation parameters, is therefore needed in order to capture the complex dynamics of the left ventricle.

3.1 Hierarchical Analysis: A Coarse-to-fine Scheme

The analysis of cardiac dynamics through shape modeling provides a compact way of characterizing the complex nonrigid motion of the left ventricle. A complete surface modeling primitive for left ventricle would require the capability to parameterize major global as well as local motion and deformations. According to Potel's findings,[11] the dynamics of left ventricle exhibits regular and periodic patterns, with apparent expansion and contraction towards a moving center. These findings suggest that there exist layers of motion and deformations which should be incorporated to construct the shape modeling primitives.

Existing methods of quantitative analysis of cardiac dynamics using shape modeling have been limited to crude estimation with simplified geometry. These simplified models capture mainly global measures of the dynamics. With a complete shape model, we expect the analysis to extend to the local measures of the dynamics that are closely related to ischemic heart diseases. Since the apparent expansion and contraction of left ventricle is towards a moving center, the identification of such moving center is very important in order to establish a reference coordinate system in which the shape modeling primitives can be defined. Such moving coordinate system can be considered as the global motion of the left ventricle, while the homogeneous expansion or contraction can be considered as the global deformations of the left ventricle. For many early studies using 2D projections or sparse 3D data, only global measures are often obtained. With volumetric data, the localized deformations can be estimated in addition to the moving center and homogeneous expansion and contraction. An one-step overall shape analysis has been proposed,[9,20] however, such one-step approach is inefficient in implementation and the estimated parameters may be inconsistent with the actual pattern of cardiac motion, since there is no *a priori* knowledge of left ventricle is incorporated in the shape model.

We present in this paper a hierarchical analysis scheme to implement the shape modeling of the left ventricle. Such scheme has been successfully applied to cine-angiographic data[1] to obtain global as well as local motion and deformations. The characterization of left ventricle dynamics through shape modeling is in a coarse-to-fine fashion. At the first level of the description, the moving center and its orientation is estimated to compute the global rigid motion of the left ventricle. The second level of description includes the expansion or contraction, bending, and tapering. Once the first level of motion is compensated, the second level of global deformation parameters can be obtained by fitting the 3D data to the superquadric global modeling primitives. The residuals of such fitting represent the localized measures that these global modeling primitives are unable to characterize. The third level of the analysis is therefore based on these residuals which can be implemented by spherical harmonics surface fitting, as will be illustrated later.

3.2 Shape Models

The global to local analysis of the left ventricle surface is based on two surfaces primitives: Superquadric surfaces and spherical harmonic surfaces. These two surface models give us the enough power to model various closed surfaces for a hierarchical fitting. According to *a priori* knowledge of left ventricle, we include the long axis bending in the superquadric modeling of the left ventricle surface. Such incorporation of the bending deformation is very effective in capturing left ventricle global deformations and results in smaller residuals to be fitted by the spherical harmonics.

Long Axis Bending: For the left ventricle, the long axis is a curve in the 3D space that lies approximately in a plane that joins the base with the apex. This curve can be estimated via a smooth function $f(z)$. We assume a isotropic bend model, where a straight line is bended to a know function preserving its length. The bending can be extended to 2D surfaces, where we assume that the principal axis of the object is bended to a known shape preserving its length, and the inner and outer sides follow the same bending function but with a varied length. Using this model the basic transformation of each point (y_s, z_s) from the bended plane to a point (d, l) in the unbended plane are:

$$l(z_i) = \int_0^{z_i} \sqrt{1 + \frac{df(z)}{dz}} dz \tag{1}$$

and

$$d = \sqrt{(y_s - f(z_i))^2 + (z_s - z_i)^2}, \tag{2}$$

where $(f(z_i), z_i)$ is the intersection point given by the shortest line connecting the surface point to the long axis curve $f(z)$.

This transformation maps each surface point vector (x_s, y_s, z_s) in the object centered coordinate system to (x_s, d, l) in the unbended coordinate system, where the x coordinate is assumed perpendicular to the bending plane and unaffected by the transformation. The motivation for such transformation is to convert the original left ventricle surface data into a new reference system in which the superquadric modeling primitives can be directly fitted to the left ventricle surface data. Without such transformation, the original surface data derived from the left ventricle chamber may not fit well with superquadrics model, since the modeling primitives are a family of symmetric surfaces while the real left ventricle surfaces exhibit some bended characteristics.

Superquadric Shape Modeling: Once we have transformed each surface point to the unbended surface points we can proceed to fit this point to superquadric modeling primitives. Superquadrics are a family of parameterized shape that have been used for shape representation in computer graphics as well in computer vision.[9] Such surfaces have the flexibility to capture the globally deformable nature of the left ventricle. A superquadric surface is the spherical product of two superquadric curves and can be defined in vector form as follows:

$$S(\theta, \phi) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_x \cos^{\epsilon_1}(\theta) \cos^{\epsilon_2}(\phi) \\ a_y \cos^{\epsilon_1}(\theta) \sin^{\epsilon_2}(\phi) \\ a_z sin^{\epsilon_1}(\theta) \end{bmatrix}, \tag{3}$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $-\pi \leq \phi \leq \pi$. Parameters θ and ϕ correspond to the latitude an longitude angles, respectively expressed in the unbended object-centered spherical coordinate system. Angle ϕ lies in the $x - y$ plane, while θ is the angle between the vector $S(\theta, \phi)$ and the $x - y$ plane. Scale parameters a_x, a_y, a_z define the size of the superquadric in the x, y and z directions, respectively, and ϵ_1 and ϵ_2 are squares parameters along the z and $x - y$ plane. By varying these parameter, superquadrics can model a large set of standard building block, such as spheres, cylinders, and parallelepipeds and shapes in between.

We can extend the superquadrics' abilities to model the left ventricle shape by introducing a tapering deformation. The tapering deformation represent a scale change in one axis. For a z-axis tapering deformation:

$$X = f_x(z)x, \quad Y = f_y(z)y, \quad Z = z, \tag{4}$$

where $f_x(z)$ and $f_y(z)$, the tapering functions, are usually piecewise linear functions of z. With this modification to the superquadric model, our global shape model is now complete. According to *a priori* knowledge, of the left ventricle, the tapering deformation can be a simple linear function of z, written as:

$$f_x(z) = k_x z + 1, \quad f_y(z) = k_y z + 1,$$

where k_x and k_y are the taperings constants along the x and y axes, respectively.

Spherical Harmonic Shape Modeling: The modeling of the unbended left ventricle shape with tapered superquadrics is still not enough to capture all the local properties of the left ventricle shape. Therefore, we need to introduce the next level of surface modeling which deals with the local shape deformations that the bended and tapered superquadric surface is unable to characterize. Such surface modeling primitives are the spherical harmonics surfaces which have the flexibility to model almost any closed shape. According to Ballard and Brown,[22] spherical harmonics are closed surfaces that and can be decomposed into a set of orthogonal functions. Spherical harmonics may be parametrized by two numbers m and n, and are continuous, orthogonal, single valued, and complete on the sphere. The basis functions $U_{m,n}$ and $V_{m,n}$ are defined in a spherical coordinate system as:

$$U_{nm}(\phi,\theta) = \cos m\phi P_{n,m}(\sin\theta) \tag{5}$$

$$V_{nm}(\phi,\theta) = \sin m\phi P_{n,m}(\sin\theta), \tag{6}$$

where $P_{n,m}$ is the Legendre function and given by:

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m}{dx^m}P_n(x), \tag{7}$$

and $P_n(x)$ is the Legendre polynomial of degree n. To represent an arbitrary shape, the radius $r(\phi,\theta)$ in a radial coordinate system can be written as a linear sum of spherical harmonics base functions:

$$r(\phi,\theta) \approx \sum_{n=0}^{N}\sum_{m=0}^{n}[A_{nm}U_{um}(\phi,\theta) + B_{nm}V_{nm}(\phi,\theta)] \tag{8}$$

In other words, any continuos surface defined on the sphere may be represented by a set of real coefficients $A_{m,n}$ and $B_{m,n}$. We choose the spherical harmonics as shape modeling primitives for the left ventricle shape and deformation analysis because they can produce closed surface with well defined functional properties. Other advantage of such surfaces is their ability to capture the details of the local surface patch without enforcing any functional constraints. Moreover, we can select the number of coefficients necessary to capture the local surfaces at a desired resolution.

4 IMPLEMENTATION WITH SURFACE FITTING

We have presented an overview of the shape estimation procedure in section 2, and the surface and bending models in section 3.2. In this section, we will describe the implementation issues in the computation of the left ventricle shape parameters. First, we analyze the object-centered coordinate system followed by the bending transformation. Then, we present the superquadric parameter estimation and finally the spherical harmonic residual fitting.

4.1 Object-centered Coordinate System

As we have discussed, a correct coordinate system would simplify the shape parameter estimation. In the case of the left ventricle, the origin of the tapered superquadric differs from its centroid. The former arises because the tapering deformation in the z axis results in a displacement of the z component of the centroid. The estimation of the object-centered coordinate system must be independent of tapering deformation. In the left ventricle case we observe that the long axis could be used to estimate this centroid because it is almost free of z axis scale change. Another advantage of using the long axis is that it is approximately bended in a plane and the estimation of the best fitting plane would give us information of the left ventricle rotations. Furthermore, we need this plane to estimate the bending function and unbend the left ventricle surface points. In order to estimate the bending plane, We must find the long axis. This axis can be estimated using the slice centroids:

$$\begin{bmatrix} x_{i,m} \\ y_{i,m} \\ z_{i,m} \end{bmatrix} = \begin{bmatrix} 1/N_i \sum_{j=1}^{N_i} x_{i,j} \\ 1/N_i \sum_{j=1}^{N_i} y_{i,j} \\ z_i \end{bmatrix} \tag{9}$$

The superquadric parameters and the rest of the shape properties are estimate more easily if we select the long axis centroid. This is so, because the main axis centroid do not carry information over the shape volume so it is a better estimator of the model origin. Thus the new reference is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/N \sum_{i=1}^{N} x_i \\ 1/N \sum_{i=1}^{N} y_i \\ 1/N \sum_{i=1}^{N} z_i \end{bmatrix} \tag{10}$$

With this centroid and the long axis points, we can compute the scattering matrix, which can be used to compute the best fitting plane. This results in the analysis of finding the best plane to fit a series of points.[21] The scatter matrix is computed using the distance of the long axis points to the LV centroid:

$$P_i = \begin{bmatrix} x_{i,m}^- \\ y_{i,m}^- \\ z_{i,m}^- \end{bmatrix} = \begin{bmatrix} x_{i,m} - x_m \\ y_{i,m} - y_m \\ z_{i,m} - z_m \end{bmatrix}, \tag{11}$$

and the scattering matrix is defined as:

$$[S] = \sum_{i=1}^{M} P_i P_i^T \tag{12}$$

Now the orientation of the normal of best fitting plane is given by the eigenvector associated with the smallest eigenvalue of the scatter matrix. At this point we have all the information to transform the surface points from the world coordinate system to the object-centered coordinate system. The new origin is given by the long axis centroid and the orientation is computed using the eigenvectors of the scatter matrix. Where the x-axis is defined as the normal to the best fitting plane and the y and z axis are estimated by the other two eigenvectors of the scatter matrix which are orthonormal to each other.

4.2 Long Axis Bending

Once we have computed the left ventricle centroid and orientation we can proceed to the surface points unbending. The surface points unbending can be done by fitting the long axis points estimated from the slice centroids. We observed that the long axis centroids resemble a cubic polynomial:

$$f_u(z) = a_o + a_1 x + a_2 x^2 + a_3 x^3.$$

With this function we are able to estimate a new coordinate system where the origin is given by the long axis centroid and the new coordinates are computed along the bended long axis by finding the point where the normal of the fitted curve intersect the surface point. This problem can be stated as a root finding problem in which we need to find the point by solving the following equation:

$$h(z_i) = -y_s - [\frac{df_u(z)}{dz}]_{z=z_i}^{-1}(z_s - z_i) + f_u(z_i) = 0 \tag{13}$$

With the intersection point $(x, f_u(z_i), z_i)$, we can compute the new coordinate system using (1) and (2). This two equations map the surface point in the object-centered coordinate system (x_s, y_s, z_s) to (x_s, d, l) in the unbended coordinate system.

4.3 Superquadric Surface Fitting

With the unbended points, we are ready for the superquadric shape fitting. Since the left ventricle resembles a ellipsoid the parameters e_1 and e_2 are fixed to 1. This problem can be stated as a non-linear optimization problem, using the inside-outside function, derived from the superquadric shape primitives. This function for the ellipsoid case is:

$$f(x,y,z) = \left((x/a_x)^2 + (y/a_y)^2\right) + (z/a_z)^2, \tag{14}$$

where if $f(x_o, y_o, z_o) = 1$, then (x_o, y_o, z_o) is on the surface; if $f(x_o, y_o, z_o) < 1$, the point is inside the surface, and if $f(x_o, y_o, z_o) > 1$ the point lies outside the superquadric surface. The objective function for the optimization problem can be defined as:

$$minimize : \sum_{i=1}^{n} |f(x,y,z) - 1|^2, \tag{15}$$

where the summation is over all the surface points.

4.4 Spherical Harmonics Surface Fitting

The next level of analysis is the estimation of the spherical harmonic coefficients. To find the coefficients we first compute the local deformation in the superquadric parametric space (θ_i, ϕ_i, r_i):

$$P_i = \begin{bmatrix} \phi_i \\ \theta_i \\ r_i \end{bmatrix} = \begin{bmatrix} \tan^{-1}(\frac{y_i'}{x_i'}) \\ \tan^{-1}(\frac{\sqrt{x_i'^2 + y_i'^2}}{z}) \\ \sqrt{x_i^2 + y_i^2 + z_i^2} \end{bmatrix}, \tag{16}$$

where

$$x_i' = \frac{x_i}{a_x(k_x z_i + 1)}, \quad , y_i' = \frac{y_i}{a_y(k_y z_i + 1)}, \quad z_i' = \frac{z_i}{a_z} \tag{17}$$

Once we have transformed the surface points to the superquadric space, we note that the spherical harmonic equation (8) can be represented as:

$$r(\phi, \theta) \approx \sum_{i=1}^{M} \alpha_i B_i(\phi, \theta). \tag{18}$$

Therefore the problem can be stated as a least squares problem, where the function to minimize is:

$$\epsilon = \sum_{j=1}^{n} \left[r_j(\phi_j, \theta_j) - \sum_{i=1}^{C} \alpha_i B_i(\phi_j, \theta_j) \right]^2. \tag{19}$$

Finally, adding the local and tampering deformations, we obtain the complete equation of the surface:

$$s(\theta, \phi) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r(\theta,\phi) f_x(z) a_x \cos^{\epsilon_1}(\theta) \cos^{\epsilon_2}(\phi) \\ r(\theta,\phi) f_y(z) a_y \cos^{e_1}(\theta) \sin^{\epsilon_2}(\phi) \\ r(\theta,\phi) a_z sin^{\epsilon_1}(\theta) \end{bmatrix}, \tag{20}$$

At this point we have completed the shape fitting, now the left ventricle shape synthesis can be carried out by taking these coefficients and applying the inverse process. The first step in the surface recovery is the recovery the local deformations of the spherical harmonic fitting. Then we add this deformation to the superquadric model. These surface points then are bended using the inverse transforms of the unbending process and finally we transform all points to the world coordinate system by inverse transforming these points from the object-centered coordinate system.

5 CURVATURE AND POINT CORRESPONDENCE RECOVERY

Motion estimation using curvature properties has already studied in Kambhamettu.[5] In the special case of conformal motion the angles between curves on the surface must be preserved. One condition that rises from this type of motion is that the ratios of the first fundamental form must be constant (t^2).[6] This means that the infinitesimal distances in the surface are stretched by a factor t at all directions at a given point. The parameter t can change from point to point; but at a given point the stretching is the same. The stretching factor t can be recovered from a given pair of surfaces, using the Gaussian curvature and the first fundamental forms of the surface.[6]

The basic idea behind Goldgof's approach is the formulation of possible point correspondences. Under the small motion assumption, the hypothesized point correspondences can be restricted to a small neighborhood around the point of interest. The correspondences are evaluated by computing the deviation from the conformal motion and the point that gives the smallest evaluation is chosen as the best point correspondence.

In this paper, we adopt the curvature approach to recover the point correspondence. We use the complete fitted surface to precompute the Gaussian curvature and the first fundamental forms. The precomputation speeds up the point correspondence recovery, and it has additional fundamental advantages over the local invariant fitting: first, our approach decompose the global motion. From the local motion so that the estimation of the point correspondence is not affected by the global motion. Second, It has been shown that over 90% of the left ventricle motion is towards the center of contraction, such *a priori* knowledge can be used to impose a constraint in the search of the point correspondence, such as a smoothness constraint. Since our surface fitting is based on a moving center of contraction, the surface smoothness constraint can be easily incorporated into parametric representation.

5.1 Recovery of the First fundamental forms and the Gaussian curvature

To recover the point correspondence, we need the coefficients of the first fundamental form and the Gaussian curvature. The first fundamental form can be easily computed using the parametric representation of the fitted surface $s(\theta, \phi)$ given by equation (20). The first fundamental form of the fitted surface is given by:

$$ds^2 = Ed\theta^2 + 2Fd\theta d\phi + Gd\phi^2, \tag{21}$$

where the coefficients E, F and G are:

$$E = (s_\theta \cdot s_\theta), \quad F = (s_\theta \cdot s_\phi), \quad G = (s_\theta \cdot s_\phi). \tag{22}$$

The Gaussian curvature can be computed using these coefficients, and their derivatives. However, they are easier to compute using the coefficients of the second fundamental form, given by:

$$L = \frac{s_{\theta\theta} \cdot s_\theta \times s_\phi}{D}, \quad G = \frac{s_{\theta\phi} \cdot s_\theta \times s_\phi}{D}, \quad L = \frac{s_{\phi\phi} \cdot s_\theta \times s_\phi}{D}, \tag{23}$$

where

$$D^2 = EG - F^2. \tag{24}$$

With the coefficients of the two fundamental forms, the Gaussian curvature is written as:

$$K = \frac{LN - M^2}{EG - F^2} \tag{25}$$

Notice that, the recovery of the Gaussian curvature and the other curvature properties can be straight forward using the estimated left ventricle surface obtained from hierarchical fitting. This approach is also able to overcome some problems caused by accumulative errors inherent in local invariant fitting.

5.2 Conformal motion matching

It has been shown, that for the conformal motion, the Gaussian curvature change is given by[6]:

$$\overline{K} = \frac{K}{t^2} + f(E, F, G, t) \tag{26}$$

where K and $\overline{K}$ represent the Gaussian curvature of the surface at a given point before and after the conformal motion, and t is the stretching factor.

From equation (26), we can define the least square error as:

$$ER = \sum_{i \in \eta} \left[\overline{K}_i - \frac{K_i}{t_i} - f(E, F, G, t) \right]^2, \tag{27}$$

where η represents the neighborhood chosen for each point on the surface for the error computation. ER can be used to compute the deviation from the conformal motion, but t is unknown. In this case we can use ER to estimate the stretching factor t that minimizes the error. In the case we assume a linear stretching the linear parameters can be obtained by equating the partial derivatives of the ER, with respect the linear parameters, to zero. These parameters minimize ER at a given point. Then the correspondence hypotheses with the minimum error give us the best point correspondence.

We incorporate a smoothness constraint weighting ER by the total deviation of the hypothesized point correspondece from a radial path. We use the parametric representation of the fitted surfaces to estimate the deviation. Given a surface point at $s(\theta_i, \phi_i)$ and its hypothesized point correspondece on the second surface point at $\overline{s}(\theta_j, \phi_j)$, we can estimate the radial deviation by computing the corresponding point separation, assuming that both points were on the second surface. That is, compute:

$$d = |\overline{s}(\theta_j, \phi_j) - \overline{s}(\theta_i, \phi_i)|, \tag{28}$$

where || represents the Euclidean distance.

Then, We define our total error as:

$$e = ER \cdot (1 + \beta d), \tag{29}$$

where β weights the importance of the distance. If β is very small the point error will follow only the estimation from the curvature properties. On the other hand, if β is big the error will be close to the total deviation from a radial displacement.

5.3 Summary of point correspondece estimation

The computation of the point correspondences using the curvature properties involves three basic steps: (1) curvature calculation and the calculation of the coefficients of the first fundamental form and its derivatives, (2) hypotheses formulation and error computation, (3) hypotheses verification.

Figure 1: a) Original left ventricle surface data extracted form the DSR b) Superquadric fitting after bending, c) 4^{th} order spherical harmonic fitting d) 10^{th} order spherical harmonic fitting

Figure 2: Point correspondence. a) frame 1 to 2. b) frame 14 to 15

The Gaussian curvature and the coefficients of the first fundamental form are computed directly from the fitted surface. In order to simplify the correspondence search, we precompute the curvature properties by sampling the fitted surface and computing all its properties at all sample points. The correspondence hypotheses are formulated using a small window around the point under consideration. The point in this window are candidates for the point correspondence. For each point we compute the estimated stretching factor and its error. The error is weighted by the deviation from the radial path. The selection of the best point correspondence is the one with the smallest error.

6 PRELIMINARY RESULTS

We have presented in section 4 the estimation process of the hierarchical surface fitting and the computation of the Gaussian curvature and the point correspondence recovery. The results of these estimations using the binary volumes segmented form a 16 time frames sequence over one cardiac cycle. Each frame consist of one volumetric image of 90x90x95 binary voxles. The original surface of frame 1 is shown in figure 1 a). Figure 1 b) shows the fitted surface to a tapered superquadric surface. Notice that the superquadric surface lacks the local modeling power and the final shape is very smooth. Figures 1 c) and 1 d) show the complete shape estimation of the left ventricle surface using 4^{th} and 10^{th} order spherical harmonic fitting.

Once we obtain the complete shape, we compute the Gaussian curvature and the coefficients of the first fundamental. Then, we proceed to the estimation of the point correspondences between frames using the conformal motion assumption. In our study we use a 11x11 window in the search of point correspondences for each point. Figures 2 a) and 2 b) shows the point correspondences between frames 1 and 2 and frames 14 to 15. The line vector indicate the point correspondence between frames.

7 CONCLUSION

We have presented an integrated hierarchical decomposition strategy to the shape and dynamics analysis of the CT volumetric images of the left ventricle. The original hierarchical decomposition was extended by incorporating *a priori* knowledge of the left ventricle, and taking advantage of the more complete surface information from the volumetric CT images. Our hierarchical shape models characterize all the global surface deformations and most of the local deformations. Such models establish a natural framework for the point correspondence recovery. Furthermore, the Gaussian curvature and other curvature properties can easily be obtained from the shape model, so that the search of the best point correspondence can be simplified. This approach can also be applied to other nonrigid motion problems when *a priori* knowledge is available and can be incorporated into the modeling primitives. More importantly, the hierarchical decomposition enables us to break a complex shape analysis into simple and computationally efficient subprocedures because our hierarchical shape analysis was based on layered shape primitives.

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